Asset Pricing Implications of Macroeconomic Interventions
An Application to Climate Policy

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ABSTRACT
This paper illustrates that evaluating alternate abatement polices that affect the growth path of an economy on the basis of their effects on asset valuation may not be welfare enhancing. We show that the class of abatement polices considered in the integrated assessment literature are robust with respect to the choice of a discount factor if lifetime consumption equivalents are used as a metric. We argue against a global welfare function in the presence of significant global household heterogeneity. While economic analysis is a useful tool for evaluating different policies for a homogenous class of households, inter household comparisons are an ethical issue.

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1. Introduction

We characterize macroeconomic interventions as interventions that change the growth path of an economy. The interventions may be due to natural causes such as the “Black Death” in the fourteenth century or the last ice age or they may be man made, such as the Great Wars in the twentieth century.

Anthropogenic climate change is an issue that has come to dominate the global public policy agenda. It is argued that if left unchecked, the accumulation of greenhouse gases in the atmosphere will negatively impact future per capita consumption. At the heart of the debate is evaluating alternative policies for greenhouse gas abatement. An abatement policy reduces current per capita consumption in exchange for a higher growth rate in the future. The default option of no intervention results in higher per capita current consumption but a lower future growth rate.

This paper illustrates that evaluating alternate abatement polices that affect the growth path of an economy based on their impact on asset valuation\(^1\) may not be a good measure of the welfare consequences of the policies\(^2\).

The framework used for evaluating alternative policies in the climate modeling literature, the various integrated assessment models, are an extension of neoclassical growth theory. Asset pricing in this context has been studied by a number of authors, including Brock (1982) and Donaldson and Mehra (1984). An insight that simplifies the analysis is the observation that the cross equation restrictions on asset prices and consumption do not depend on whether the

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\(^1\) Dell et al (2012) examine the effects of temperature shocks on economic growth. For a discussion of the impact of climate change on asset valuation see Ahlroth (2009) and Hanewinkel (2013) and the references in these papers

consumption process is endogenously determined, as in an economy with
production or is exogenous as in an exchange economy\(^3\). The relative prices of the
date, event-contingent, composite-consumption goods are proportional to the
marginal rates of substitution of consumption. The deus ex machina of this class
of asset pricing models is that for a given endowment process, household trading
of financial assets is motivated by a desire to smooth consumption, both over
time and across states at a point in time. The desirability of an asset in this
model reflects its ability to smooth consumption. Hence, assets that pay off in
future states when consumption levels are high – when the marginal utility of
consumption is low – are therefore less desirable than those that pay an
equivalent amount in future states when consumption levels are low and
additional consumption is more highly valued\(^4\). As a consequence, the price of a
claim to a unit of consumption at some future time \(t\) scales in proportion to the
marginal utility of consumption at that time. Both the household’s elasticity of
intertemporal substitution and risk aversion play a crucial role in this class of
models.

In these models, the price of an asset at time \(t\), \(p_t\) with stochastic payoffs
\(\{y_s\}_{s=t+1}^\infty\) is

\[
p_t = E \left[ \sum_{s=t+1}^\infty m_{s,t} y_s | \Phi_t \right]
\]  

(1)

where \(\{m_{s,t}\}_{s=t+1}^\infty\) a stochastic process\(^5\), \(\Phi_t\) is the information available to
households who trade assets at time \(t\) and \(E\) is the expectations operator. \(m_{s,t}\) is

\(^3\) In a production economy the class of consumption processes will be a subset of the processes in
an exchange economy. See Mehra and Prescott (2008) Appendix C.

\(^4\) Consumption levels are relative to current consumption.

\(^5\) \(m_{s,t} = \prod_{k=0}^{s-t-1} m_{t+k+1,t+s} \); where \(m_{t+k+1,t+s}\) is a random variable such that
usually expressed as a function of the marginal rate of substitution of consumption between time $s$ and $t$ of the agents who trade securities. For example, in a widely cited and influential paper, Lucas (1978) models $m_{s,t}$ as

$$\beta^{s-t}u'(c_s) / u'(c_t).$$

Here $c_t$ is the aggregate per capita consumption at time $t$, $u'(c_t)$ is the marginal utility of consumption at time $t$ and $\beta$ is the rate of time preference. In the case of isoelastic utility, $m_{s,t}$ specializes to $\beta^{s-t}(c_s / c_t)^{-\alpha}$, where $\alpha$ is the coefficient of relative risk aversion and, simultaneously, the reciprocal of the elasticity of intertemporal substitution.

In this model class, Mehra and Prescott (1985) find that the *premium for bearing non-diversifiable aggregate risk is small*. This premium$^6$ is approximately equal to $\alpha \sigma_c^2$ where $\sigma_c^2$ is the variance of the *growth rate* of consumption. For the U.S. the historical average for $\sigma_c^2$ is 0.0018. Since plausible values of $\alpha$ are upper bounded by 10, we will abstract from this small premium and consider an economy with no aggregate uncertainty. Uncertainty is not essential to the point we establish and this abstraction simplifies the analysis$^7$.

The point of departure of this paper is the observation that any major intervention in the economy, such as climate abatement policies, is likely to alter the growth rate of consumption, *hence relative prices and discount rates, will not be invariant to such policy interventions*$^8$.

A change in the growth rate of consumption will change the stochastic process $\{m_{s,t}\}_{s=t+1}^\infty$ thereby changing prices relative to current consumption of all assets in the economy. The intuition is straightforward. A reduction in the

$$p_{s+k} = E \left[ m_{s+k+1} n_{s+k+1} \mid \Phi_{s+k} \right].$$

$^6$ It is exact if the growth rate of consumption is i.i.d and log-normally distributed.

$^7$ The issues involved in the treatment of uncertainty, including uncertainties about the magnitude, timing, and impacts of climate change, are discussed in Howarth (2003), Weitzman (2007, 2009) and Ackerman et al. (2009).

$^8$ See Lucas (1976) for a related observation.
growth rate of an economy, will in steady state, have two effects. It will reduce future consumption and it will lead to a reduction in the equilibrium interest rate. The first effect tends to reduce the relative value while the second increases the value of assets relative to current consumption. The net result is ambiguous. We illustrate this in section 2.

The paper consists of 5 sections. In section 3 we evaluate a stylized abatement policy based on two measures: asset valuation and household welfare, measured in lifetime consumption equivalents and show that these lead to inconsistent rankings. In section 4 we address heterogeneity in preferences and issues of aggregation. Section 5 concludes the paper.

2. An Illustrative Example

Consider a certainty analog of an endowment economy of the type analyzed in Mehra and Prescott (1985). There is a single infinitely lived household with isoelastic preferences. This unit orders its preferences over consumption paths by

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\alpha} - 1}{1 - \alpha}, \quad 0 < \beta < 1, \quad 0 < \alpha < \infty$$

where \(c_t\) is the per capita consumption.

The parameter \(\beta\) is the rate of time preference, which describes how impatient households are to consume. If \(\beta\) is small, people are highly impatient, with a strong preference for consumption now versus consumption in the future. The parameter \(\alpha\) measures the curvature of the utility function. When \(\alpha = 1\), the utility function is defined to be logarithmic, which is the limit of the above

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9 Relative to an economy with no reduction in its growth rate.
representation as \( \alpha \) approaches 1.

As modeled, the household lives forever, which implicitly means that the utility of parents depends on the utility of their children\(^{10} \).

Land is the only asset in this economy. Its ownership entitles the owner to the entire consumption stream \( \{c_s\}_{s=t+1}^{\infty} \). The price of land at time \( t \), \( p_t \) is\(^{11} \):

\[
p_t = \sum_{s=t+1}^{\infty} \beta^{s-t}(c_s / c_t)^{-\alpha} c_s
\] (3)

The discount factor is the sequence \( \{m_{s,t}\}_{s=t+1}^{\infty} \) with \( m_{s,t} = \beta^{s-t}(c_s / c_t)^{-\alpha} \).

Consider two such economies, identical in every respect expect that land in one economy produces the consumption good with a growth rate of 2\% while in the other the growth rate is 1\%. The household elasticity of intertemporal substitution is 0.5 (\( \alpha = 2 \)) and its \( \beta = 0.99 \). Consumption levels are 1 in both economies at time 1.

Let \( p^h_0 \) and \( p^l_0 \) be the price of land in the high growth and the low growth economies respectively at time 0. Using the pricing relation above, it is straightforward to find relative valuation of the land in the two economies in today’s consumption equivalent.

\[
p^h_0 / p^l_0 = \frac{(1 + g^h)^{\alpha - 1} - \beta}{(1 + g^l)^{\alpha - 1} - \beta}
\] (4)

For an economy characterized by the above parameters we find that \( p^h_0 / p^l_0 = 2 / 3 \) even though the household welfare is higher in the higher growth economy irrespective of \( \alpha \). The reason for this counter-intuitive result is that,

\(^{10}\) See Becker and Barro (1988)
\(^{11}\) See equation (6) in Mehra and Prescott (1985). As shown in Mehra (1988), equilibrium in this economy can exist even if \( \beta > 1 \). Equilibrium will exist if \( \beta < (1 + g)^{\alpha - 1} \). We assume that this condition holds in this paper.
as mentioned earlier, the discount rate is a function of the growth rate of consumption and is different in the two economies. The discount rate is given by:\(^{12}\):

\[ r = -\ln \beta + \alpha \mu_c \]  

(5)

where \( \mu_c = \ln(1 + g) \) is the continuously compounded growth rate of consumption. For the U.S, over the past 100 years the growth rate of consumption has averaged slightly less than 2%. With \( \alpha = 2 \), in the high growth economy the discount rate is 5% while in the low growth economy it is 3%. In the low growth economy the increase in the value of land due to the lower discount rate more than offsets the reduction due to the decrease in consumption. In the case when household \( \alpha \) is 1 (logarithmic preferences) \( \frac{p_0^h}{p_0^l} = 1 \), as the two effects exactly offset each other.

Table 1 reports the value of \( \frac{p_0^h}{p_0^l} \) and confirms this observation for a wide range of values of \( \alpha \) and \( \beta \).

This simple example illustrates that evaluating policies that affect aggregate consumption based on their impact on asset values may not be a good measure of the welfare consequences of the policies.

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\(^{12}\) See equation 18 in Mehra (2012). Note that we are assuming no uncertainty, hence \( \sigma_c^2 = 0 \). Tol (1994) discusses the case when \( \mu_c \) is negative.
### Table 1

Values for $p_h^b / p_0^l$ implied by $\alpha$ and $\beta$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>0.97</th>
<th>0.98</th>
<th>0.99</th>
<th>1.0</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td></td>
<td>1.24</td>
<td>1.48</td>
<td>34.15</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.80</td>
<td>0.75</td>
<td>0.67</td>
<td>0.50</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.71</td>
<td>0.66</td>
<td>0.60</td>
<td>0.50</td>
<td>0.33</td>
</tr>
</tbody>
</table>

N.A: Equilibrium does not exist.

### 3. Evaluation of an GHG Abatement Policy

In this section, we evaluate a stylized abatement policy based on two distinct measures: asset valuation and household welfare. We remind the reader that in the integrated assessment models of the type analyzed by Stern (2007 and 2008) and Nordhaus (2008), the welfare consequences and asset valuations depend only on the endogenous consumption process. The policy analyzed below can be considered as a “reduced form” of an integrated assessment model with an equivalent consumption process. The parameters we consider encompass a wide variety of current policy recommendations. Pindyck (2011,2012) has forcefully argued that abatement policies should be modeled as effecting the growth rate of consumption rather than its level. This paper endorses this perspective and expands on it.

We evaluate two alternative policies in an economy of the type analyzed in Section 2 above:

a) No abatement or intervention: in this scenario, per capita
consumption initially grows at the historical average rate of 2% for $T$ years. Thereafter, due to the effects of greenhouse gas emissions, the growth rate slows down to 1% and remains at that level in perpetuity\textsuperscript{13}. The reasons for this slow-down cited in the literature include: conflicts, pandemics, large scale flooding and the consequent migration of a vast number of people.

b) An abatement policy, which requires investment in carbon abatement technologies for $T$ years, resulting in a reduction in current per capita consumption by $x\%$ per year during the investment period. This investment is in addition to the normal investment required for a 2% growth (in the absence of GHG emissions). However, as a result of the abatement the growth rate remains constant at 2% indefinitely.

Figure 1 illustrates the per capita consumption path over time in the two cases.

\textsuperscript{13} The results will not change if instead of a reduction in perpetuity, the reduction was for, say, 100 - 150 years. The present value of a long annuity is well approximated by a perpetuity for the discount rates considered in Table 2.
We compute the present value of land and the lifetime consumption equivalents (defined below) with and without abatement for a range of the parameters $\alpha$, $\beta$, and $g$. As shown in Table 2 these parameter values encompass a wide range of discount rates $r$ defined in equation 5.
Table 2

Values for $r$ (%) implied by $\alpha$ and $\beta$ when $g = 2\%$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>0.98</th>
<th>0.99</th>
<th>0.999</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>N.A</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Values for $r$ (%) implied by $\alpha$ and $\beta$ when $g = 1\%$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>0.98</th>
<th>0.99</th>
<th>0.999</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>N.A</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

N.A: Equilibrium does not exist.

We reiterate that the land entitles the owner to the entire future consumption stream $\{c_s\}_{s=t+1}$ and hence its price is equal to the present value of this consumption stream, valued at the endogenous interest rate. To compute the present value of land with no abatement, $p_0^{na}$, we modify equation 2 to account for the decrease in growth rate after time $T$.

$$p_0^{na} = \sum_{t=1}^{T} \beta^t (c^h / c_0)^{-\alpha} c_t^h + \sum_{t=T+1}^{\infty} \beta^t (c^l / c_0)^{-\alpha} c_t^l$$

(6)

where

$$c_t^h = c_0 (1 + g^h)^t \quad 1 \leq t \leq T$$
\[ c_t^l = c_0 (1 + g^h)^T (1 + g^l)^{t - T} \quad T + 1 \leq t \]

To compute the present value the land under abatement \( p_{0wa} \) we use

\[ p_{0wa} = \sum_{t=1}^{T} \beta^t (c_t^h / c_0)^{-\alpha} c_t^h (1 - x) + \sum_{t=T+1}^{\infty} \beta^t (c_t^h / c_0)^{-\alpha} c_t^h \quad (7) \]

where

\[ c_t^h = c_0 (1 + g^h)^t \quad 1 \leq t \]

Table 3 reports\(^\text{14}\) the values of \( p_{0wa} / p_{0na} \) for \( \alpha = 2 , \, \beta = 0.98 \) \( x \in \{1\%,2\%,3\%\} \) and \( T \in \{50,100,150,\infty\} \) (the case \( T = \infty \) serves as an integrity check).

**Table 3**

<table>
<thead>
<tr>
<th>( T )</th>
<th>( x% )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.949</td>
<td>0.940</td>
<td>0.932</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.984</td>
<td>0.974</td>
<td>0.964</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>0.989</td>
<td>0.979</td>
<td>0.969</td>
<td></td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.990</td>
<td>0.980</td>
<td>0.970</td>
<td></td>
</tr>
</tbody>
</table>

While utility is a monotonic transformation of consumption, this transformation is usually nonlinear. The present value of consumption is not the same as the present value of lifetime utility and in general, is not the correct measure of welfare.

\(^{14}\) In the Appendix A we report the results for a range of parameters \( \alpha \in \{1,2,3\} \) and \( \beta \in \{0.98,0.999\} \).
While the various integrated assessment models do base their calculations on the maximization of lifetime utility, abatement and non-abatement in these models effect the levels rather than the growth rates of consumption as in our analysis. This explains why our conclusions are different from these models.

To calculate the welfare impact of the different policies, we compute the lifetime consumption equivalents\(^ {15} \) as in Lucas (1987). Using the no abatement policy as the base, this is the fraction of consumption that would make a household indifferent between the two policies.

Define lifetime utility with no abatement \( U_{0}^{na} \) as:

\[
U_{0}^{na} = \sum_{t=1}^{T} \beta^t \left( \frac{(c_{t}^h)^{1-\alpha}}{1-\alpha} - 1 \right) + \sum_{t=T+1}^{\infty} \beta^t \left( \frac{(c_{t}^i)^{1-\alpha}}{1-\alpha} - 1 \right) \tag{8}
\]

where

\[
c_{t}^h = c_{0}(1 + g^h)^t \quad 1 \leq t \leq T
\]

\[
c_{t}^i = c_{0}(1 + g^h)^T (1 + g^i)^{t-T} \quad T + 1 \leq t
\]

and compensated lifetime utility with abatement \( U_{0}^{wa}(\lambda) \) as:

\[
U_{0}^{wa}(\lambda) = \sum_{t=1}^{T} \beta^t \left( \frac{(c_{t}^h(1 - x)(1 + \lambda))^{1-\alpha}}{1-\alpha} - 1 \right) + \sum_{t=T+1}^{\infty} \beta^t \left( \frac{(c_{t}^i(1 + \lambda))^{1-\alpha}}{1-\alpha} - 1 \right) \tag{9}
\]

where

\[
c_{t}^h = c_{0}(1 + g^h)^t \quad 1 \leq t
\]

\(^ {15} \) The astute reader would have noticed that we cannot take the ratio of lifetime utility of consumption to evaluate policies. For instance a utility of -30 > -50 but 3/5 is less than 1. Nor is the magnitude of the differences in utility an indicator of a quantitative welfare improvement, as utility functions can be arbitrarily scaled without changing the preference orderings. Computing lifetime consumption equivalents makes it possible to make quantitative assessments for a homogenous class of households.
If we equate $U_0^{sa}(\lambda)$ to $U_0^{na}$ and solve for $\lambda$, we get the fraction by which consumption under abatement must be increased or decreased to make the household indifferent to the no abatement policy.

Table 4 reports the values of $\lambda$ for $\alpha = 2$, $\beta = 0.98$, $x \in \{1\%, 2\%, 3\%\}$ and $T \in \{50, 100, 150, \infty\}$ (the case $T = \infty$ serves as an integrity check).

Table 4

<table>
<thead>
<tr>
<th>$T$</th>
<th>$x%$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td>-0.034</td>
<td>-0.024</td>
<td>-0.014</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>0.004</td>
<td>0.014</td>
<td>0.025</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>0.009</td>
<td>0.020</td>
<td>0.030</td>
</tr>
<tr>
<td>$\infty$</td>
<td></td>
<td>0.010</td>
<td>0.020</td>
<td>0.031</td>
</tr>
</tbody>
</table>

For the values of the parameters $\alpha$ and $\beta$ considered in tables 3, 4 and in appendices A and B, if $T=50$ years, a policy maker would not choose the abatement policy based on market valuation even though it is welfare enhancing in each instance. The lifetime consumption equivalents are negative. Households are willing to sacrifice consumption, in addition to that required by the abatement policy, to avoid a reduction in future growth rates.

A striking result of this analysis is that the policy recommendations are robust across a wide range of discount rates ranging from 3% to 6%. Abatement is always welfare enhancing if $T=50$ years, while it is almost neutral in terms of lifetime consumption equivalents even if $T=100$ years. This is in sharp contrast

\[16\] In the Appendix B we report the results for a range of parameters $\alpha \in \{1, 2, 3\}$ and $\beta \in \{0.98, 0.999\}$. 

to conventional wisdom\textsuperscript{17} and to the differing recommendations in Stern (2008) and Nordhaus (2008).

We reiterate, that the underlying debate should not be about which discount rate to use\textsuperscript{18} in evaluating alternative policies, but rather on the effect of the policies on the growth rate of the economy. \textit{The discount rate is endogenous.}

It has been argued in the environmental literature that the welfare of future generations should not be valued less than that of the present generation\textsuperscript{19}. In our view, inter generational discounting is an ethical issue to which economic theory provides little guidance\textsuperscript{20}. In tables 5 and 6 we consider a case with $\beta > 1$; its conclusions are consistent with our earlier observations -- there is no striking difference between tables 3 and 4 and tables 5 and 6.

\begin{table}[h]
\centering
\begin{tabular}{llll}
\hline
\textbf{PV with abatement/PV with no abatement} & \textit{p_0^{\text{w}}} / \textit{p_0^{\text{n}}} \\
\textbf{\textit{\alpha = 3, \beta = 1.01}} & & & \\
\hline
\textbf{\textit{T}} & \textbf{\textit{x\%}} & \textbf{1} & \textbf{2} & \textbf{3} \\
50 & 0.681 & 0.670 & 0.670 \\
100 & 0.897 & 0.889 & 0.880 \\
150 & 0.967 & 0.958 & 0.948 \\
\infty & 0.990 & 0.980 & 0.970 \\
\hline
\end{tabular}
\caption{Table 5}
\end{table}

\textsuperscript{17} See Weitzman (2007). “The biggest uncertainty of all in the economics of climate change is the uncertainty about which interest rate to use for discounting”.


\textsuperscript{19} This position is supported by Broome (2008), Cline (2006), Cowen (2007), Dasgupta (2008), Heal (2009), Philibert (1999), Rawls (1999), Ramsey (1931) and Sidgwick (1890) among others. See the review by Revesz and Shahabian (2011).

\textsuperscript{20} In contrast, for intra generational discounting, a plausible case can be made for $\beta < 1$. See Arrow (1999) or Blanchard (1985).
Table 6

Lifetime consumption equivalent $\lambda$

$\alpha = 3$, $\beta = 1.01$

<table>
<thead>
<tr>
<th>$T$</th>
<th>$x%$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td>-0.163</td>
<td>-0.154</td>
<td>-0.146</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>-0.039</td>
<td>-0.029</td>
<td>-0.019</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>-0.002</td>
<td>0.009</td>
<td>0.019</td>
</tr>
<tr>
<td>$\infty$</td>
<td></td>
<td>0.010</td>
<td>0.020</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Given that the integrated assessment models abstract from a positive feedback induced catastrophe, our analysis favors a policy of abatement as an inexpensive (in terms of lifetime consumption equivalents) insurance policy, given the present modeling perspective\textsuperscript{21}.

This recommendation may be strengthened in a model that incorporates endogenous technological change and substitution between technologies that will be an inevitable consequence of relative prices changes in the factors of production.

We hasten to add that we are not taking a stand on the type of abatement investment and the conclusions of our analysis do not depend on this. It may well be the case that increased investment for R&D in abatement technology today and then the use of this more efficient technology for abatement in the future may be more effective in curtailing greenhouse emissions than with existing technology.

\textsuperscript{21} If, as shown above, abatement is approximately welfare neutral in models that abstracts from catastrophic risk, it will be welfare enhancing if we include and price this risk. The increase in welfare arises because abatement offers the benefits of a put option that we have not included.
4. Household Heterogeneity

Most of the climate literature assumes a “global representative household”. The reality, however, is that there is significant household heterogeneity across the globe. Large parts of the population in India, China and sub-Saharan Africa live at or near subsistence levels of consumption. This group accounts for about a third of global households, and their willingness to substitute consumption over time is arguably different from that of households in more developed economies. Lending rates for poorer households are, very likely, much higher than those implied by capital market data.

To illustrate this, consider a preference function of the form:

\[ u(c_t, \bar{c}) = c_t - \bar{c}^{1-\alpha} - 1 \]

(10)

where \( \bar{c} \) is the subsistence level of consumption.

For these preferences the relative risk aversion is:

\[ \frac{-c_t u''(c_t)}{u'(c_t)} = \frac{\alpha}{1 - \bar{c}/c_t} \]

(11)

Rich households have consumption levels well above subsistence levels and \( \bar{c}/c_t \) for this class of households is likely to be small and their risk aversion with be approximately equal to \( \alpha \). Poor households on the other hand are likely to have consumption levels closer to subsistence levels and the household’s effective (or
local) risk aversion becomes very large. For example, if $\alpha = 2$ and $\bar{\sigma}/c_i \approx 0.9$ then the effective risk aversion of these households $\approx 20!$

How does one deal with household heterogeneity\textsuperscript{22}? Unfortunately, as is well known, a social welfare function cannot be constructed in general if household preferences are heterogeneous. If the heterogeneous households have preferences that satisfy the conditions for aggregation\textsuperscript{23}, then a representative agent can be constructed in a manner that is independent of the underlying heterogeneous agent economy’s initial wealth distribution. Although aggregation permits the use of the representative agent for welfare comparisons, it substantially narrows the choice of utility functions. While the CRRA preferences considered in this paper satisfy the conditions for aggregation, Epstein-Zin preferences do not\textsuperscript{24}. Any welfare analysis using the later preference class is effectively based on the preferences of a “global dictator”. Even if preferences are of the CRRA form there is no general closed form representation that relates the heterogeneity in $\alpha$ at the household level to the curvature of the representative agent.

Economists can evaluate the impact of a policy on the welfare of each heterogeneous class of agents. \textit{Weighing the interests of different classes is an ethical issue and in general is outside the scope of economics.}

5. Conclusion

This paper illustrates that even in a homogenous agent economy, using social discount rates for evaluating alternative abatement policies on the basis of its effects on asset valuation may not be welfare enhancing when these policies affect the growth rate of consumption.

\textsuperscript{22} See the recent working paper by Hassler and Kursell (2012).
\textsuperscript{23} Households need to have common homothetic preferences. See Acemoglu (2008) chapter 5.
\textsuperscript{24} Epstein and Zin (1991).
We show that the class of abatement polices considered in the integrated assessment literature are robust with respect to the choice of a discount factor if lifetime consumption equivalents are used as a metric.

We argue against a global welfare function in the presence of significant global household heterogeneity. While economic analysis is a useful tool for evaluating different policies for a homogenous class of households, inter household comparisons are an ethical issue.
References


Pindyck R.S. Modeling the Impact of Warming in Climate Change Economics, Chapter 2 in The Economics of Climate Change, University of Chicago Press, 2011.


Sidgwick H. The Method Of Ethics. Oxford University.1890


## Appendix A

### PV with abatement / PV with no abatement $p_0^{\text{wa}} / p_0^{\text{na}}$

$\alpha = 1$, $\beta = 0.98$

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\text{PV with abatement/PV with no abatement } P_0^{\text{wa}} / P_0^{\text{na}} \\
\alpha &= 2, \quad \beta = 0.999
\end{align*}
\]

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\alpha &= 3, \quad \beta = 1.01
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Appendix B

**Lifetime consumption equivalent $\lambda$**

$\alpha = 1$, $\beta = 0.98$

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**Lifetime consumption equivalent $\lambda$**

$\alpha = 2$, $\beta = 0.98$

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**Lifetime consumption equivalent $\lambda$**

$\alpha = 1$, $\beta = 0.99$

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**Lifetime consumption equivalent \( \lambda \)**

\[ \alpha = 2, \quad \beta = 0.999 \]

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**Lifetime consumption equivalent \( \lambda \)**

\[ \alpha = 3, \quad \beta = 1.01 \]

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